Unit 5: Analyze, Solve, and Graph Linear Inequalities

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Unit 5: Analyze, Solve, and Graph Linear Inequalities

Instructor Notes

The Mathematics of Linear Inequalities

This unit introduces inequalities. The mathematics of solving linear inequalities is so similar to that of linear equations that most students will have little trouble with the calculations. There are a number of conceptual challenges, however. Let's tackle them one learning objective at a time.

Teaching Tips: Writing, Solving and Graphing Inequalities in One Variable

Lesson 1, Topic 1 has two objectives:

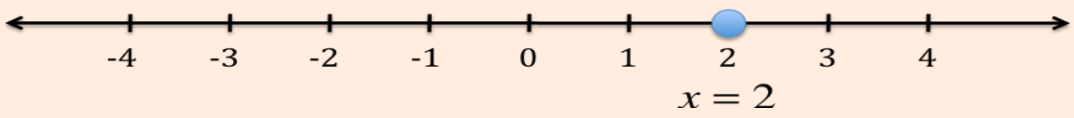
- *Solve algebraic inequalities in one variable using a combination of the properties of inequality*
- *Represent inequalities on a number line*

Challenge: Students have difficulty understanding why they need to reverse the direction of an inequality when multiplying or dividing both sides by a negative number.

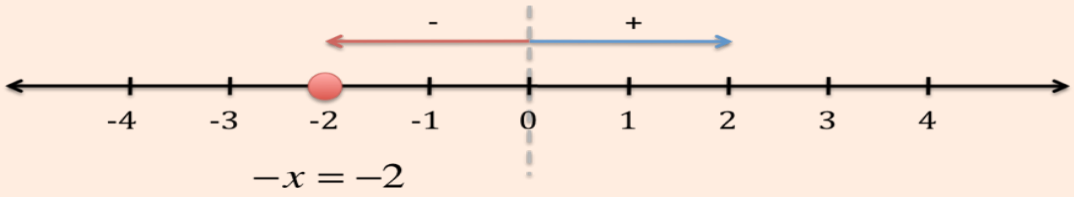
Remedy: Use number lines to help students get a better understanding of what negative really means.

Example

Graph an equation like $x = 2$ on the number line:



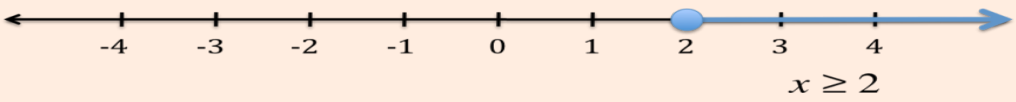
Multiply both sides of the equation by a negative number, say -1. Now $-x = -2$. Describe how multiplying by -1 is the same as flipping (or reflecting) the whole number line around the origin, so the new graph is:



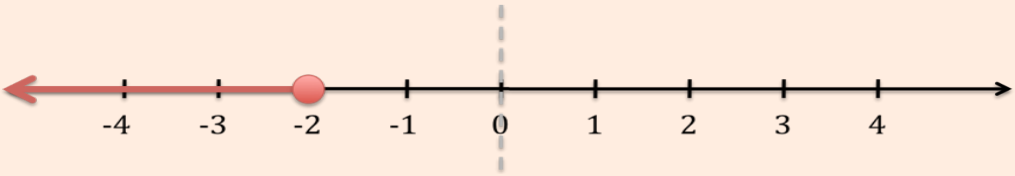
Make sure students see that multiplying by a negative number has moved the point to the opposite side of zero.

Then repeat the process with an inequality instead of an equation.

Here's the graph of $x \geq 2$:



If we multiply this inequality by a negative number like -1, the graph has to flip over to the opposite side of the origin just like it did before:



To correctly describe the new graph, we have to flip the inequality sign and write $-x \leq -2$. Multiplying both sides by -1 will give us *–x* on the left and -2 on the right. But now the arrow points to the left instead of the right. The "greater than" inequality has become a "less than" inequality.

This visual approach will help to give students an intuitive sense of why the inequality sign flips, but it is important to back this up with a little simple algebra:

$$x \geq 2$$
$$0 \geq 2 - x$$
$$-2 \geq -x$$
$$-x \leq -2$$

We've just shown that if $x \geq 2$, then $-x \leq -2$. Although we got there by subtraction, this is equivalent to multiplying by -1. When we multiply an inequality by a negative number, we must change the direction of the inequality sign.

Teaching Tips: Solving and Graphing Absolute Value Inequalities

Lesson 1, Topic 2 has two objectives:

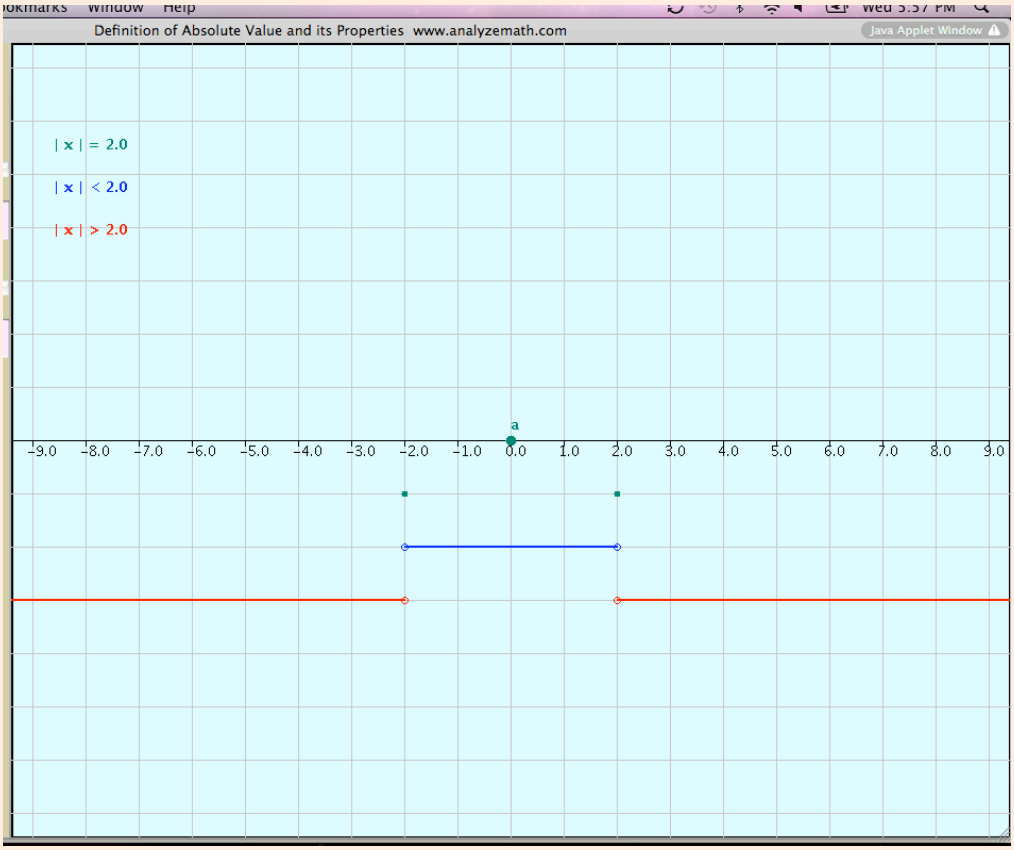
- *Solve absolute value inequalities in one variable using a combination of the properties of inequality*
- *Represent absolute value inequalities on a number line*

Challenge: Students will struggle with the idea that absolute value inequalities have two solutions. For example, given the inequality $|x+2|>5$, many students will decide that $x>3$. While it may seem as though they are simply forgetting the other possibility, in many cases students just don't understand the absolute value concept.

Remedy: Use manipulatives to conduct a classroom discussion about what graphs of absolute value inequalities look like. Then split the class into small groups to produce their own graphs of a variety of absolute value inequalities.

Example

The graph below plots the solutions to |*x*| = 2, |*x*| < 2 , and |*x*| > 2. Hands-on practice finding and comparing simple inequalities like these will help students grasp the meaning and impact of the absolute value sign.



This graph was produced using the tools at http://www.analyzemath.com/Definition-Absolute-Value/Definition-Absolute-Value.html. Other free online programs, such as Geogebra, are also available.

Teaching Tips: Writing and Using Inequalities

Lesson 1, Topic 3 has 1 objective:

- *Develop inequalities to represent real world situations and use them to solve problems*

Challenge: The hardest part of turning word problems into inequalities is the vocabulary. For example, the subtle differences between conditions like "2 more than *x*", "2 is more than *x*", and "2 is no more than *x*" can make it difficult for students to translate such verbal descriptions into accurate inequality statements.

Remedy: It's harder to clue in to vocabulary when the context is strange or outside a student's experience. Although working through a lot of word problems can help, mere repetition isn't enough. Students will be far more likely to internalize the vocabulary of inequalities when they use it for themselves in a meaningful context.

Example

The project for Unit 5, "Open for Business" lets students apply what they have learned in a context they create largely themselves (they have to develop a business model and decide where to price items for sale and determine how this will affect profits). When students are ready to present their projects, you could provide a list of relevant sentence starters in advance such as "The profit is more than…" and "The price can be no more than…" etc., and require them to use these in their presentations. You could also include similar smaller listening and speaking activities throughout the unit. This will ensure that they get practice at using this vocabulary in a meaningful context.

Teaching Tips: Solving and Graphing Linear Inequalities in Two Variables

Lesson 2, Topic 1 has 2 objectives:

- *Represent linear inequalities as regions on the coordinate plane*
- *Determine if a given point is a solution of a linear inequality*

Challenge: Students have difficulty linking algebra to the geometry of the coordinate plane.

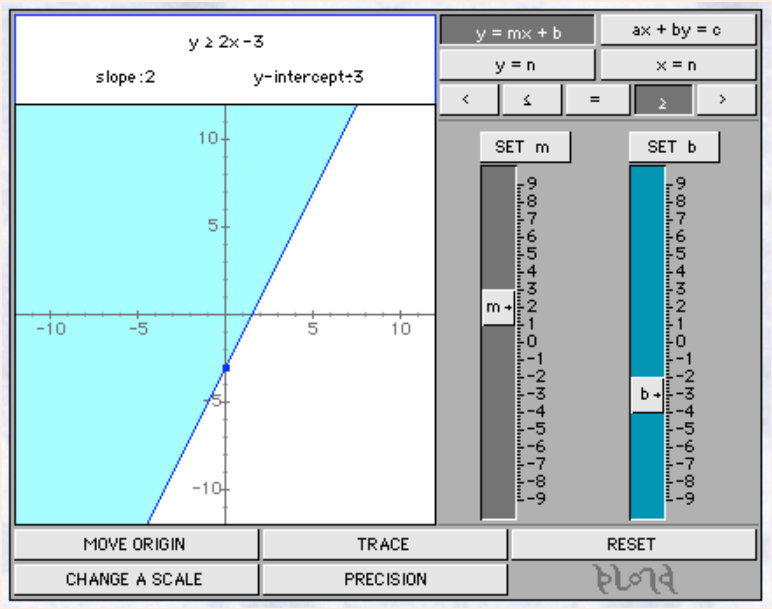
Remedy: Students need to follow the logic that an algebraic statement such as $y_1 < mx_1 + b$ defines a particular region of the coordinate plane, and that points in that

region are solutions to the inequality. Hands on manipulatives help students visualize the geometry, and see if a given point is inside or outside a given region on the plane. Use programs like Geogebra or the applet at http://www.ronblond.com/M11/LinIne/index.html to graph and work through two variable inequalities, then assign students further problems to practice on their own.

Example

Pose a problem like "Determine whether the point (2, 5) is a solution of the inequality $y \geq 2x - 3$."

First, have a student find (2, 5) on the coordinate plane (if you have an interactive whiteboard you could plot the point over the top of the applet). Next, use the applet to graph the inequality $y \geq 2x - 3$.



It will be plain that the point (2, 5) does lie inside the shaded area, and therefore is a solution of the inequality. Finally, have all the students plug the coordinates (2, 5) in for (*x*, *y*) in the inequality $y \geq 2x - 3$. When they find that $5 \geq 1$, explain that since the inequality is true, the solution is valid. This will help the students build the crucial connections they need between the algebraic and visual representations of inequalities.

Summary

This unit teaches students how to work with linear inequalities. Although the mathematics are familiar by now, inequalities are different enough from equations to pose some conceptual difficulties. These can be overcome through the use of number lines, manipulatives, and the use of word problems that put inequalities into familiar situations.

Unit 5: Analyze, Solve, and Graph Linear Inequalities

Instructor Overview
Tutor Simulation: Skateboarding Championship

Purpose

This simulation is designed to challenge a student's overall knowledge of linear functions and inequalities. Students are asked to apply what they have learned to solve a real-world problem by demonstrating an understanding of the following areas:

- Patterns
- Domain and Range
- Linear Functions
- Writing Equations and Inequalities
- Inequalities
- Graphing

Problem

Students are given the following problem:

> We're sending you on an all-expense paid trip to see the elimination round of the State Skateboarding Championships. Unfortunately, by the time we made your trip plans, all the tickets were sold. Fortunately, we got you into the event, but you'll have to help the judges calculate and analyze the scores.
>
> Your challenge will be to analyze the scores and help the judges decide who gets to continue to the final round.

Recommendations

Tutor simulations are designed to give students a chance to assess their understanding of unit material in a personal, risk-free situation. Before directing students to the simulation,

- make sure they have completed all other unit material.
- explain the mechanics of tutor simulations
    - Students will be given a problem and then guided through its solution by a video tutor;
    - After each answer is chosen, students should wait for tutor feedback before continuing;
    - After the simulation is completed, students will be given an assessment of their efforts. If areas of concern are found, the students should review unit materials or seek help from their instructor.
- emphasize that this is an exploration, not an exam.

Unit 5: Analyze, Solve, and Graph Linear Inequalities

Instructor Overview
Puzzle: What's More

Objective

What's More is a puzzle that asks players to assess the weights of fruits on balance scales. Students must create and compare inequalities in order to choose the heaviest fruit.

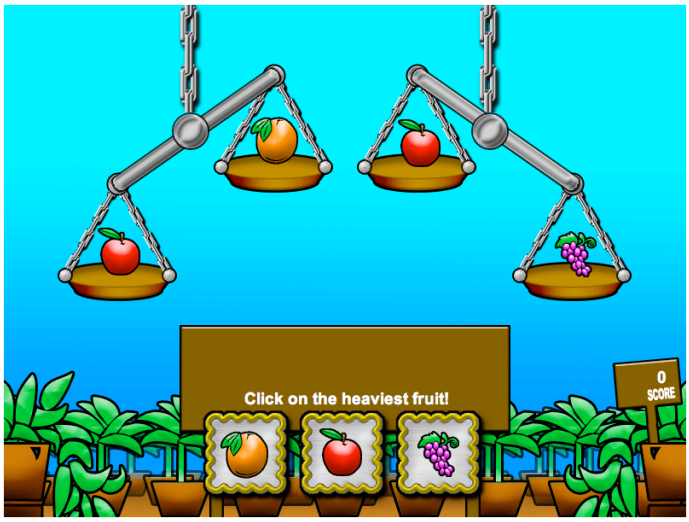


Figure 1. *What's More? offers an unbalanced diet of fruit so that the learner can practice working with inequalities.*

Description

In each level, players see multiple scales holding various fruits. They have to identify which fruit weighs more on each scale, and then combine all the individual inequalities to find out which fruit is the heaviest of them all. Once the learner has chosen the heaviest fruit, the inequality representing the relationship of the fruits is shown to reinforce the learner's analysis. Players score points for solving the puzzles and the points accumulate across puzzles. There are three levels of difficulty. In level one, there are two scales and three fruits. In level two, four fruits are arranged on three scales. In level three, four fruits are grouped in combinations on three scales.

There are ten puzzles at each level that are generated in real time, so each level can be played over and over. *What's More?* is suitable for individual play and could also be used in a classroom setting to provoke interest in the topic and to allow learners to articulate their reasoning in determining their solution.

Unit 5: Analyze, Solve, and Graph Linear Inequalities

Instructor Overview
Project: Open for Business!

Student Instructions

Introduction
Have you ever wondered what it would be like to run your own business? Business owners have to make decisions every day in order to maximize their profits (the money they make) and increase their number of customers. If they price their product too low, they will have lots of customers but will make less money per product sold. If they price their product too high, they will have fewer customers willing to pay the high price, but will make more money per unit. Businesses are constantly researching and evaluating their pricing structure in order to stay competitive in the marketplace.

Task
During the course of this project, your team will choose products to sell, conduct market research to set the prices, and evaluate the potential profits of a well-planned business. First, you will choose two products that you would like to sell while keeping in mind your customer base (other students in your class or your school). Next, you will need to research your cost in acquiring the products that you will sell. After that, you will survey your classmates and group members to determine an acceptable price for your product. Finally, you will write a mathematical model to represent your profit based upon the number of products sold.

Instructions
Solve each problem in order, keeping careful notes along the way. From your notes, you will create a final presentation to share with your classmates.

1 First problem:
- What two products would your group like to sell? Think about your customer base (your class). What do students your age like to spend their extra money on? You may want to take a survey of your class in order to gather more information.

2 Second problem:
- What will the two products cost you to buy? Research the prices online. Determine your cost per unit based on your research. Try the following websites for general merchandise, candy, or food products:
  http://www.toyconnection.com/
  www.sweetservices.com
  http://www.wholesalecentral.com/Food-Grocery.html

3 Third problem:

- Write and graph an inequality statement for the possible selling prices of each product. Graph the possible selling prices of each product on its own number line.

  (Hint: Is there a minimum price for each product? How do you determine the minimum price? What is the maximum selling price? It is okay to try a very high price for now. You will collect data to set your reasonable price next.)

4 Fourth problem:

- Make a function table to represent total profit as a function of the number of items sold. You should have four function tables, two for your first product and two for your second product. One function table for each product should represent pricing your item at the minimum level and the other will represent maximum pricing.

- Now graph the data for your first product on one coordinate plane and the data for your second product on another. You should have two lines on each coordinate plane.

- How are the two lines similar? How are they different? Calculate the slope of each of the four lines. What does the slope of each line represent? Notice that the area between the two lines represents the possible profit per item. This profit will fluctuate based upon your choice in pricing. You will look at determining an appropriate price next.

5 Fifth problem:

- Create a poll to test your selling prices. Space your prices out between the minimum and maximum prices to determine what your classmates would be willing to pay for each of your two products. You can conduct a poll in your classroom or set up an online poll using a site such as:
  www.polldaddy.com
  www.surveymonkey.com

6 Sixth problem

- Based on the results of the poll, determine your maximum reasonable price for each product. Remember, if you set your prices too high, no one will choose to buy your products. Be prepared to discuss how you chose this price during your presentation.

7 Seventh problem

- Determine your profit per unit for your first product, x. For this project, your profit per unit will be the cost per unit subtracted from the selling price per unit. Next, determine your profit per unit for your second product, y. Your business will be considered

successful if you can profit at least $100. Using your profit per unit x, unit y, and the $100 goal, write and graph a linear inequality. Use the model below to write your inequality.

$$\boxed{\text{Profit per unit x}} \cdot x + \boxed{\text{Profit per unit y}} \cdot y \geq 100$$

(Hint for the graph: Think back to standard form. How many of only unit *x* would you have to sell to make your goal? How many of only unit *y*? There are also multiple combinations of *x* and *y*. How do you show this on your graph?)

- Based on your graph, give three possible combinations of product *x* and product *y* sold in order to meet or exceed your goal. Explain your reasoning for choosing each combination and mark each point that was chosen on your graph.

Collaboration
Discuss the benefits and drawbacks of the various types of businesses. Take a class vote to determine which business would be the best to try to run within your school. Is it possible to actually run your business before or after school? Who would you need to contact for approval? Consider giving your profits to charity or using them for a class party.

Conclusions
Determine the best way to share your project with your class. You will need to share your solutions to each of the problems, as well as the necessary graphs. You will also want to either have high quality pictures of your products or the actual products to show to the class. Be sure to support each decision that you made with data from your class surveys or from your Internet research. Your options for the presentation could include: a multimedia presentation, a project board, or an oral presentation.

Instructor Notes

Assignment Procedures

Problem 4
Make sure each group is creating proper function tables, like the sample below. Check that they are calculating slope correctly and that the slope of each line is related to the profit per item.

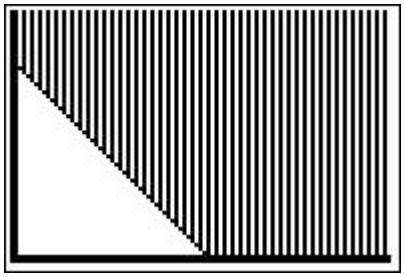
Minimum Price Bubblegum		Maximum Price Bubblegum	
# of Items	Total Profit	# of Items	Total Profit
0	0	0	0
10	1.50	10	5.00
20	3.00	20	10.00
30	4.50	30	15.00
40	6.00	40	20.00

Problem 5
Both of the suggested poll creation websites are free to use and are quite easy to set up. If an online poll is not practical, students can survey using traditional paper and pencil methods.

Problem 6
This is a great opportunity to discuss pricing with the students. Why is it that a merchant can charge $3.75 for a bottle of water at an amusement park? How much does a bottle of water cost at the grocery store? What are the reasons for the differences in pricing?

Problem 7
Remind the students how to graph inequalities and identify which region of the coordinate plane satisfies an inequality.

The students should create a graph similar to the one below, where the number of product *x* sold is plotted against the number sold of product *y*. Any point within the shaded area would represent earning at least $100 in profit. It is important for the students to understand what any ordered pair on the graph represents. For example, (50, 40) might represent 50 packages of bubble gum and 40 stickers.



Recommendations:

- have students work in teams to encourage brainstorming and cooperative learning.
- assign a specific timeline for completion of the project that includes milestone dates.
- provide students feedback as they complete each milestone.
- ensure that each member of student groups has a specific job.

Technology Integration

This project provides abundant opportunities for technology integration, and gives students the chance to research and collaborate using online technology. The following are examples of free internet resources that can be used to support this project:

http://www.moodle.org
An Open Source Course Management System (CMS), also known as a Learning Management System (LMS) or a Virtual Learning Environment (VLE). Moodle has

become very popular among educators around the world as a tool for creating online dynamic websites for their students.

http://www.wikispaces.com/site/for/teachers or http://pbworks.com/content/edu+overview
Lets you create a secure online Wiki workspace in about 60 seconds. Encourage classroom participation with interactive Wiki pages that students can view and edit from any computer. Share class resources and completed student work with parents.

http://www.docs.google.com
Allows a student to collaborate in real-time from any computer. Google Docs provides free access and storage for word processing, spreadsheets, presentations, and surveys. This is ideal for group projects.

http://why.openoffice.org/
The leading open-source office software suite for word processing, spreadsheets, presentations, graphics, databases and more. It can read and write files from other common office software packages like Microsoft Word or Excel and MacWorks. It can be downloaded and used completely free of charge for any purpose.

Rubric

Score	Content	Presentation
4	Your project appropriately answers each of the problems, shows the steps taken, and the rationale behind each of your group's decisions. Your project appropriately uses single variable inequality statements and graphs to represent the possible prices of each product. Your project also uses a two variable inequality statement and graph to represent the possible sales to meet the profit goal.	Your presentation contains information shown in a logical and interesting sequence that is easy to follow. Professional looking visual aids add to the interest and help the audience connect with your presentation.
3	Your project appropriately answers each of the problems and shows the steps necessary to solving each problem. Your project uses single variable inequality statements and graphs to represent the possible prices of each product. Your project also uses a two variable inequality statement and graph to represent the possible sales to meet the profit goal. (A few minor errors in the inequalities	Your presentation contains information presented in a logical sequence that is easy to follow. Some visual aids are used to help promote interest in the presentation.

	may be present.)	
2	Your project minimally answers each problem, but does not show the steps taken to answer each problem. An attempt is made to write and graph the necessary inequality statements, but major errors are noted.	Your presentation is hard to follow because the material is presented in a manner that jumps around between unconnected topics. Some low quality visual aids are used but do not add interest to the presentation.
1	Some problems are unsolved and/or the steps taken to answer each problem are not shown. Inequality statements and graphs are missing or inadequate.	Your presentation is difficult to understand because there is no sequence of information. Visual aids are missing.

Unit 5: Algebra— Analyze, Solve, and Graph Linear Inequalities

Glossary

absolute value	the value of a number without regard to its sign
boundary line	a line that represents the edge of a linear inequality: if points along the boundary line are included in the solution set, then a solid line is used; if points along the boundary line are not included in the solution set, then a dashed line is used
bounded region	the set of solutions that are true for all of the linear inequalities under consideration
coordinate plane	a plane in two dimensions, containing the x- and y-axes, used to map ordered pairs in the form (x, y)
inequality	a math sentence that defines a range of numbers; inequalities contain the symbols <, ≤, >, or ≥
linear inequality	an inequality represented in a form equivalent to Ax + By > C, where the symbol > could also be <, ≤, or ≥
Properties of Inequality	a set of rules for inequalities that describe how addition, subtraction, multiplication, or division can be applied to both sides of an inequality in order to produce an equivalent inequality
ray	a half-line beginning at one point and continuing to infinity
slope-intercept form	a linear equation, written in the form y = mx + b, where m is the slope and b is the y-intercept

NROC Algebra 1--An Open Course
Unit 5
Mapped to Common Core State Standards, Mathematics

Algebra 1 | Analyze, Solve, and Graph Linear Inequalities | Writing, Solving and Graphing Inequalities in One Variable | Writing, Solving and Graphing Inequalities in One Variable

Grade: 7 - Adopted 2010

STRAND / DOMAIN	**CC.7.EE.**	Expressions and Equations
CATEGORY / CLUSTER		Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
STANDARD	**7.EE.4.**	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
EXPECTATION	**7.EE.4.b.**	Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.

Grade: 9-12 - Adopted 2010

STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-REI.**	Reasoning with Equations and Inequalities
STANDARD		Solve equations and inequalities in one variable.
EXPECTATION	**A-REI.3.**	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Algebra 1 | Analyze, Solve, and Graph Linear Inequalities | Writing, Solving and Graphing Inequalities in One Variable | Solving and Graphing Absolute Value Inequalities

No Correlations

Algebra 1 | Analyze, Solve, and Graph Linear Inequalities | Writing, Solving and Graphing Inequalities in One Variable | Writing and Using Inequalities

Grade: 7 - Adopted 2010

STRAND / DOMAIN	**CC.7.EE.**	Expressions and Equations
CATEGORY / CLUSTER		Use properties of operations to generate equivalent expressions.

STANDARD	**7.EE.2.**	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."
STRAND / DOMAIN	**CC.7.EE.**	Expressions and Equations
CATEGORY / CLUSTER		Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
STANDARD	**7.EE.4.**	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
EXPECTATION	**7.EE.4.b.**	Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.

Grade: 9-12 - Adopted 2010

STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-CED.**	Creating Equations
STANDARD		Create equations that describe numbers or relationships.
EXPECTATION	**A-CED.1.**	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
EXPECTATION	**A-CED.3.**	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-REI.**	Reasoning with Equations and Inequalities
STANDARD		Solve equations and inequalities in one variable.
EXPECTATION	**A-REI.3.**	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Algebra 1 | Analyze, Solve, and Graph Linear Inequalities | Solving and Graphing Linear Inequalities in Two Variables | Solving and Graphing Linear Inequalities in Two Variables

Grade: 9-12 - Adopted 2010

STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-REI.**	Reasoning with Equations and Inequalities

STANDARD		Represent and solve equations and inequalities graphically.
EXPECTATION	**A-REI.12.**	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.